Constrained Bimanual Planning with Analytic Inverse Kinematics

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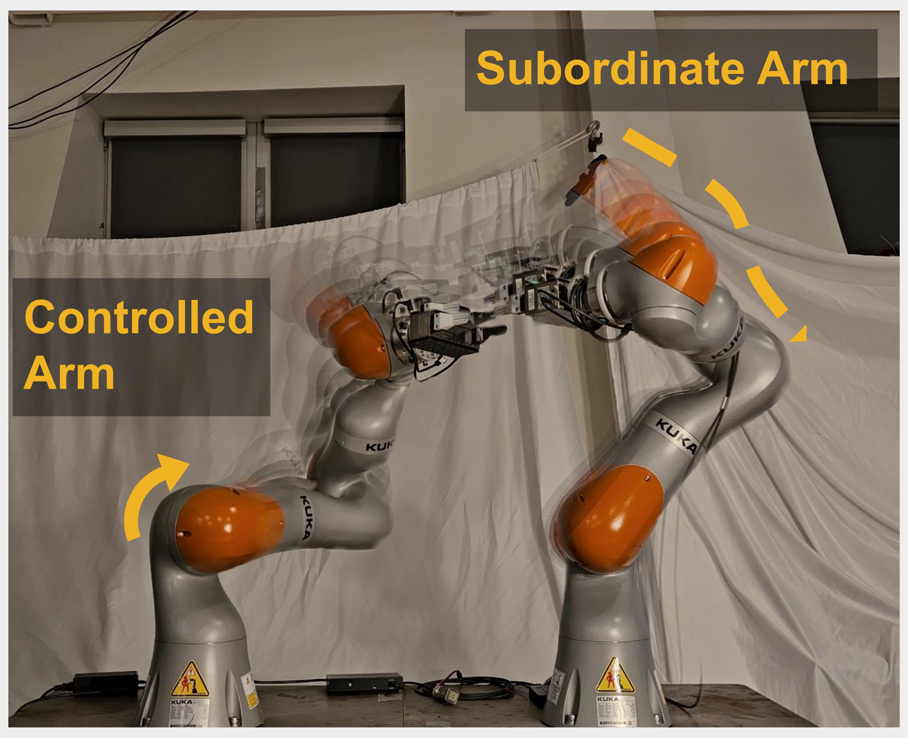
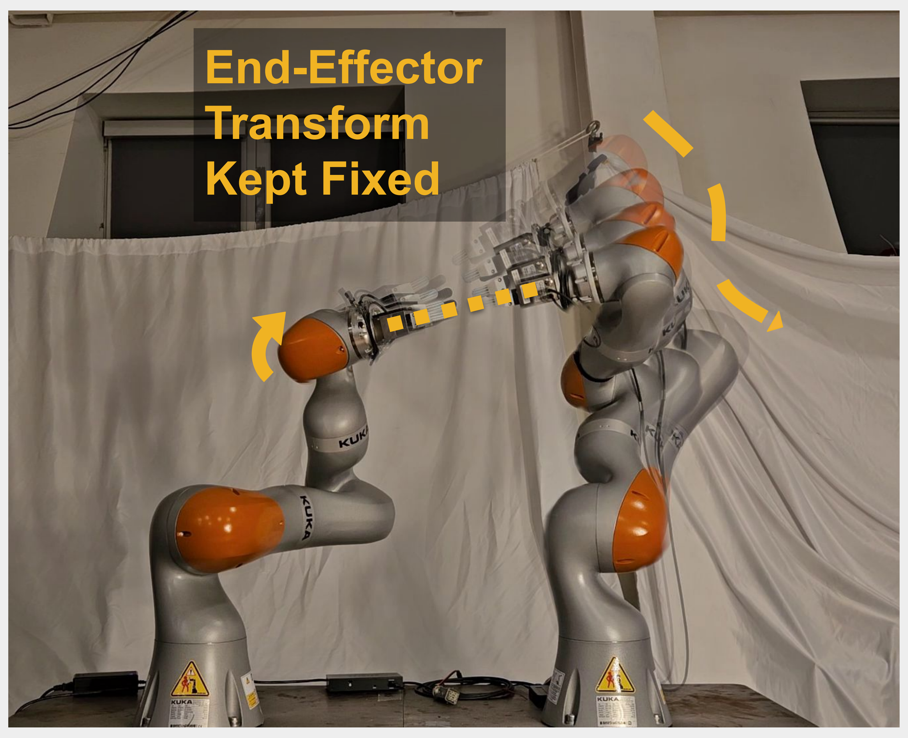
We present a minimal coordinates parametrization for bimanual manipulators with relative end-effector pose constraints, such as when the two hands are manipulating a rigid object.

Why Use this Parametrization?

- *Automatically satisfy kinematic constraints*
- *Works with any planning method*
- *No nonlinear equality constraints*
- *Differentiate through the IK mapping to use trajopt*

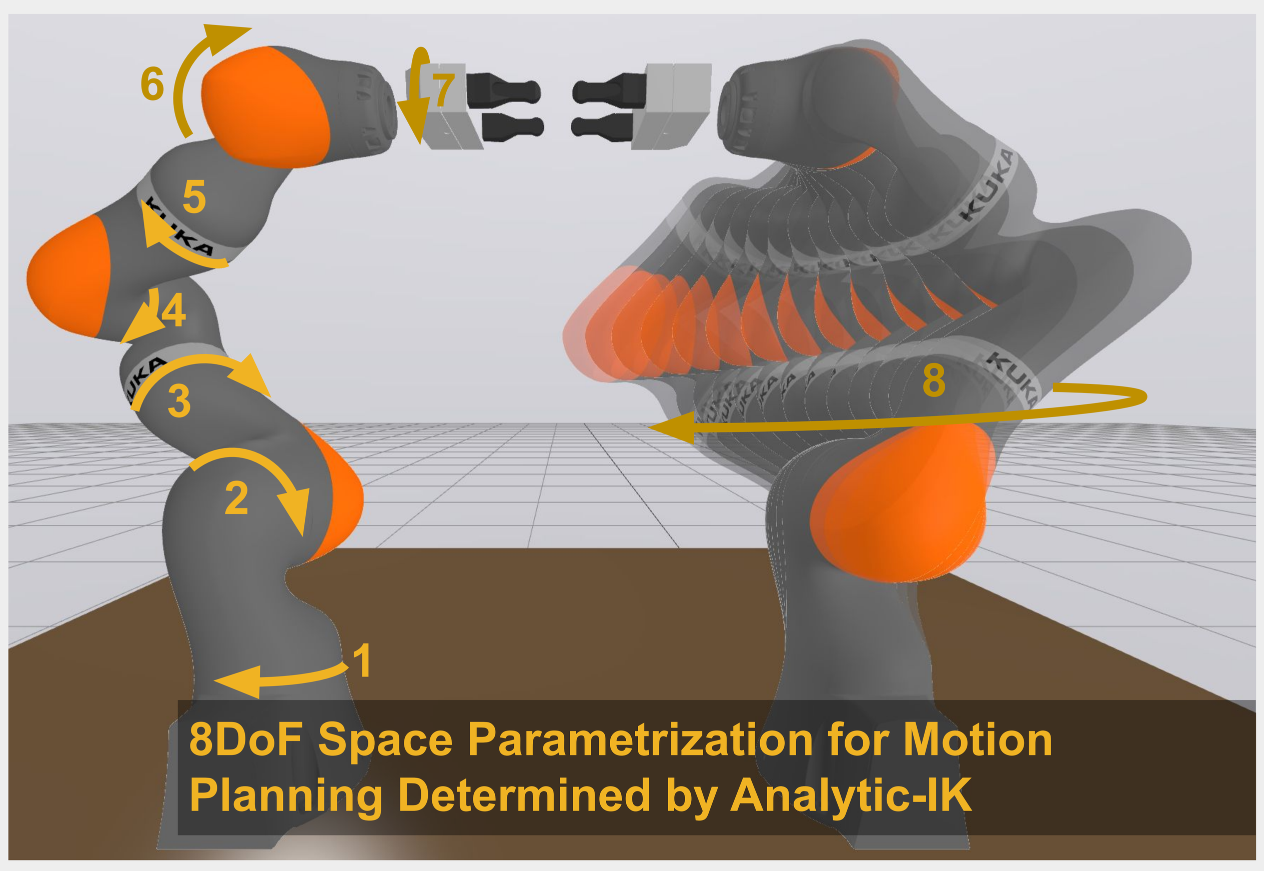
Parametrization



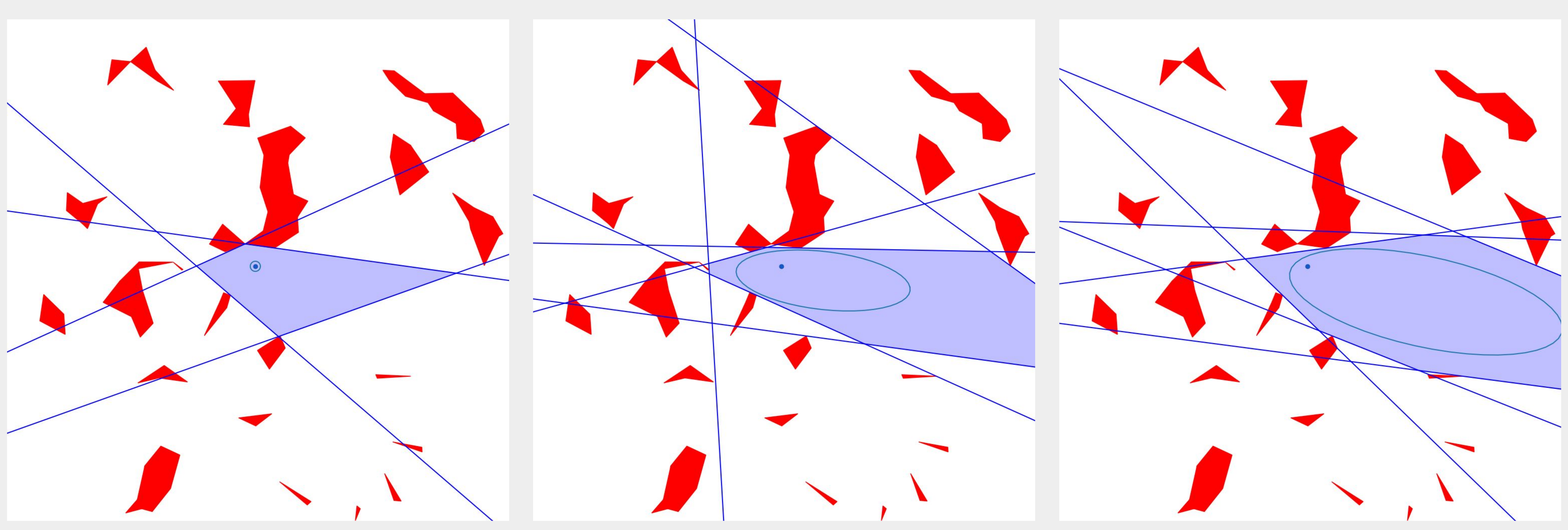


Planning is done in the configuration space of the joints of the *controlled arm* and the continuous redundancy parameter of an analytic IK solution applied to the *subordinate arm*.

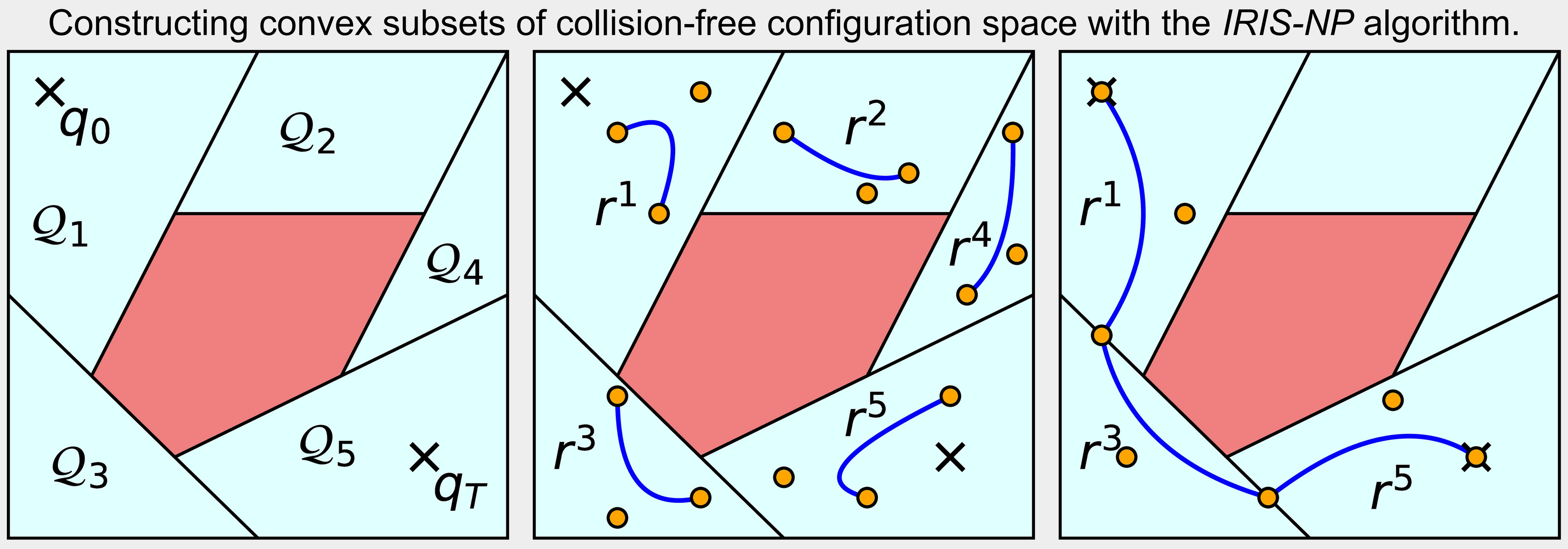
Individual Degrees of Freedom



Planning through Convex Sets



Constructing convex subsets of collision-free configuration space with the *IRIS-NP* algorithm.



Reproduced from *Motion Planning around Obstacles with Convex Optimization*, Marcucci et. al.

Growing Convex Sets

Algorithm 1: Constrained IRIS (Single Iteration)

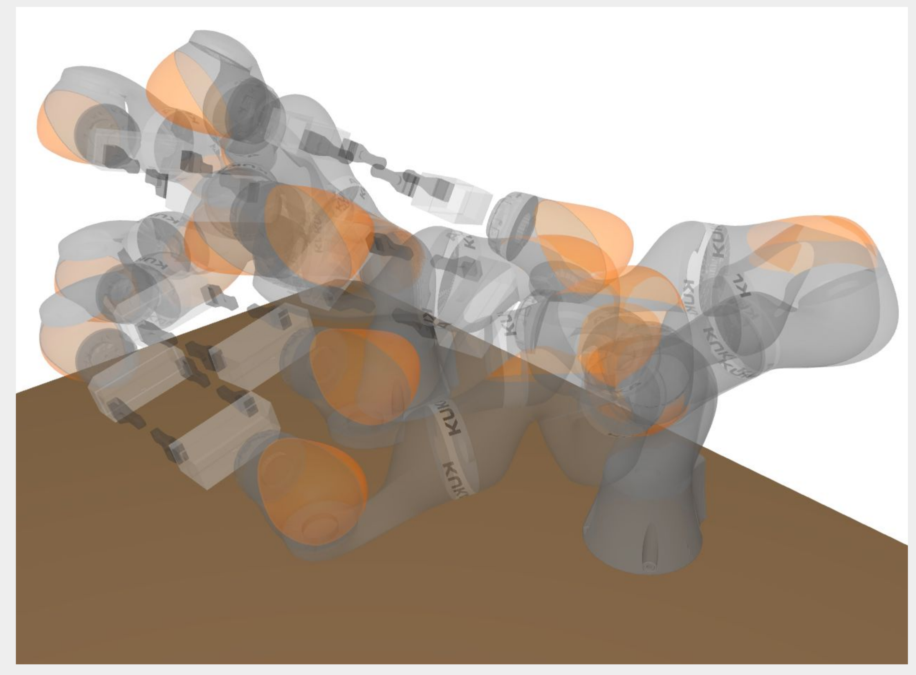
Input: Bounding Box $\mathcal{H}_0(A_0, b_0)$
Hyperellipsoid $\mathcal{E}(C, d)$ s.t. $d \in \mathcal{H}_0(A_0, b_0)$
Constraint Sets $\mathsf{CS}_1, \ldots, \mathsf{CS}_k$
Output: Halfspace Intersection $\mathcal{H}(A, b)$

```
1 A ← A_0, b ← b_0
2 for CS = CS_1, ..., CS_k do
3     repeat
4         (a*, b*) ← SOLVE[(6), {A, b, C, d, CS}]
5         A ← VSTACK(A, a*), b ← VSTACK(b, b*)
6     until INFEASIBLE
7 return H(A, b)
```

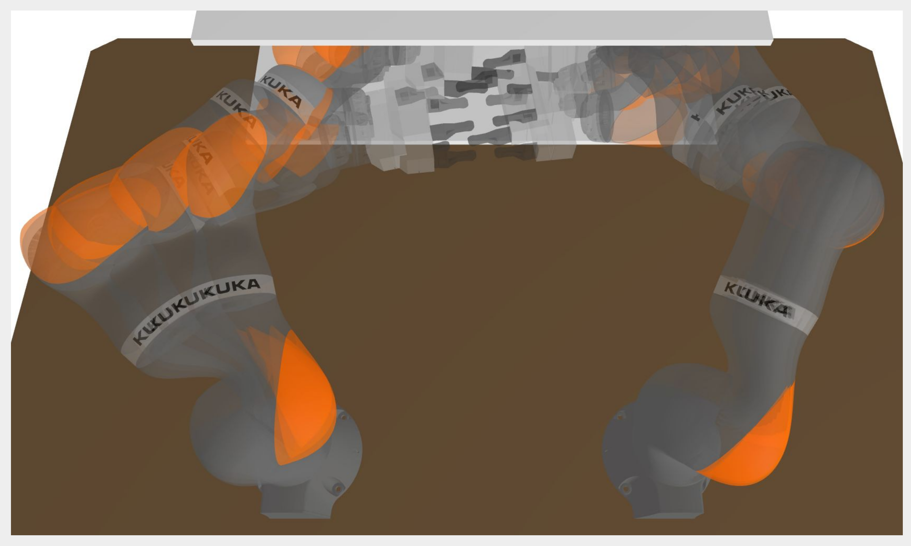
Counterexample Search Program (6)

$$\begin{aligned} \min_q \quad & \|q - d\|_C^2 \\ \text{s.t.} \quad & Aq \leq b \\ & q \notin \mathsf{CS} \end{aligned}$$

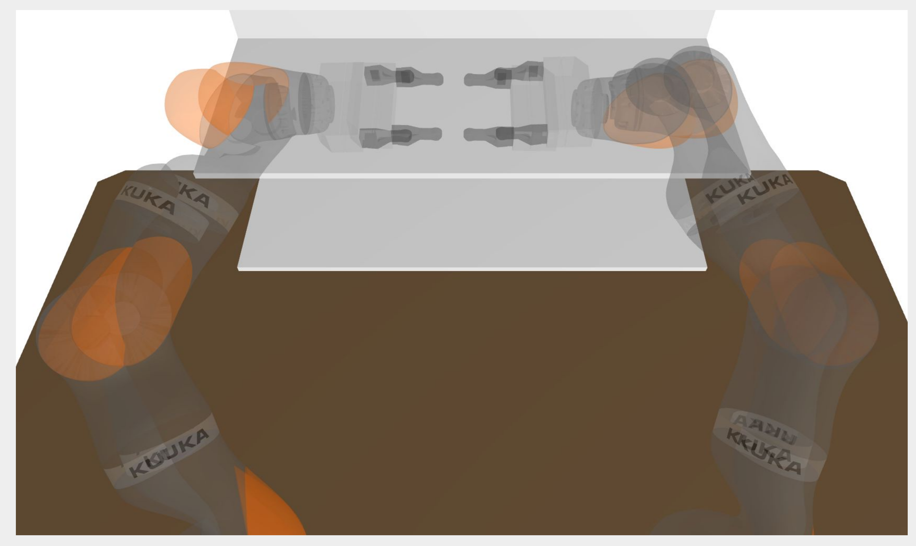
We visualize a C-space convex set by plotting multiple configurations contained within it.



Individual sets cover large parts of task space.

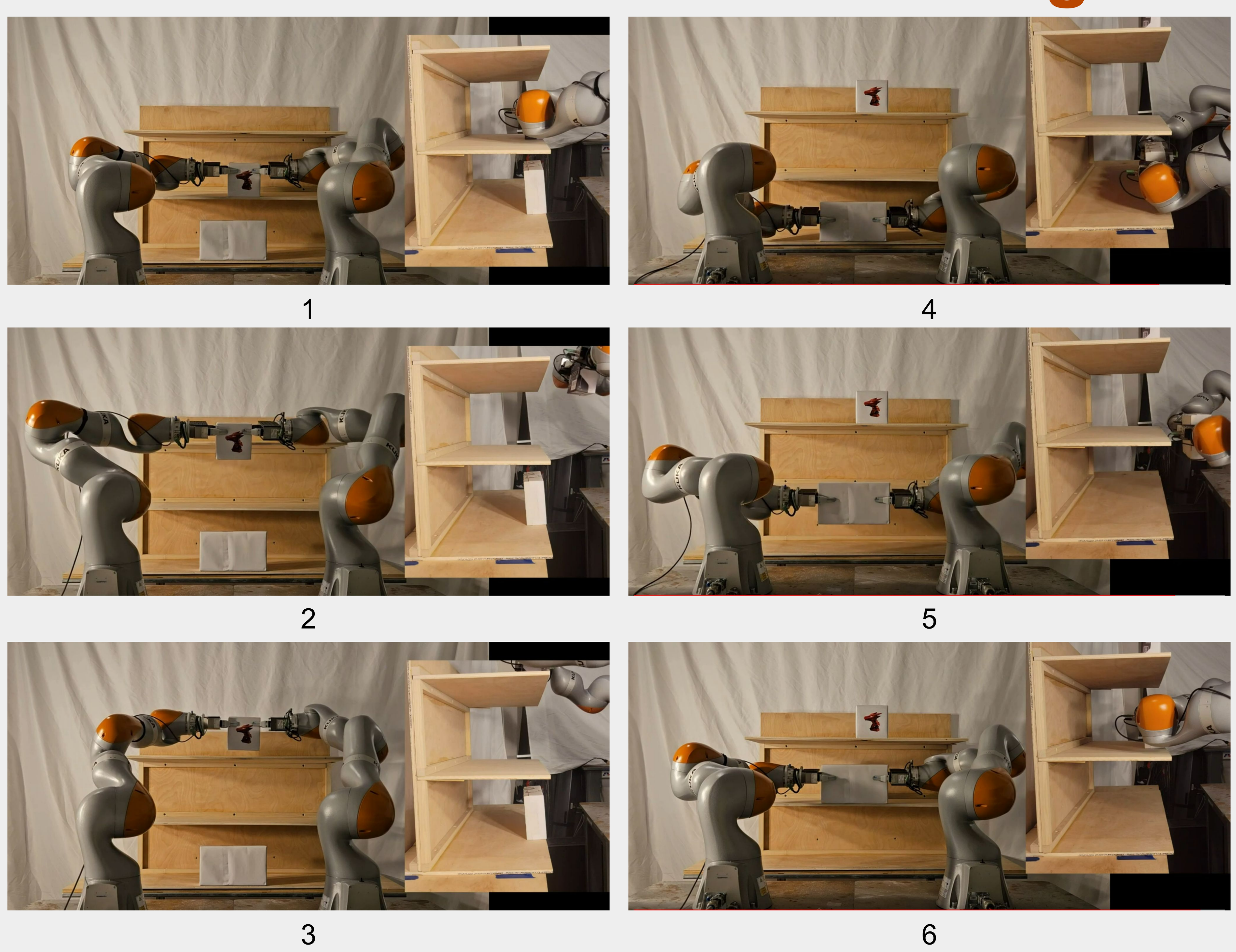


Individual sets allow a robot to reach into and out of a set of shelves.



Individual sets can treat a varying grasp distance as an additional degree of freedom.

Bimanual Motion Planning



Moving boxes of different sizes between a set of shelves with the *Graph of Convex Sets* planner.

Comparison of Algorithms

Method	Top to Middle	Middle to Bottom	Bottom to Top
Trajopt	4.58*	2.85*	**4.35***
Atlas-BiRRT	4.72	5.04	6.61
Atlas-PRM	5.43	5.67	6.99
IK-Trajopt	4.24*	**1.81***	8.87
IK-BiRRT	9.91	8.69	11.42
IK-PRM	4.67	8.93	9.21
IK-GCS	**2.09**	3.32	5.62

Comparison of path lengths (in configuration space) for different planning approaches.

- Asterisks denote plans that had collisions.
- BiRRT plans are averaged over 10 runs.

Method	Top to Middle	Middle to Bottom	Bottom to Top
Trajopt	10.37	5.36	7.25
Atlas-BiRRT	140.82	155.91	201.32
Atlas-PRM	0.69	0.86	0.96
IK-Trajopt	19.48	18.70	22.29
IK-BiRRT	49.42	52.53	54.10
IK-PRM	**0.46**	**0.64**	**0.61**
IK-GCS	3.41	2.32	3.32

Comparison of online planning runtimes (in seconds).

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