A Framework for Combining Optimization-Based and Analytic Inverse Kinematics

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Optimization Inverse Kinematics

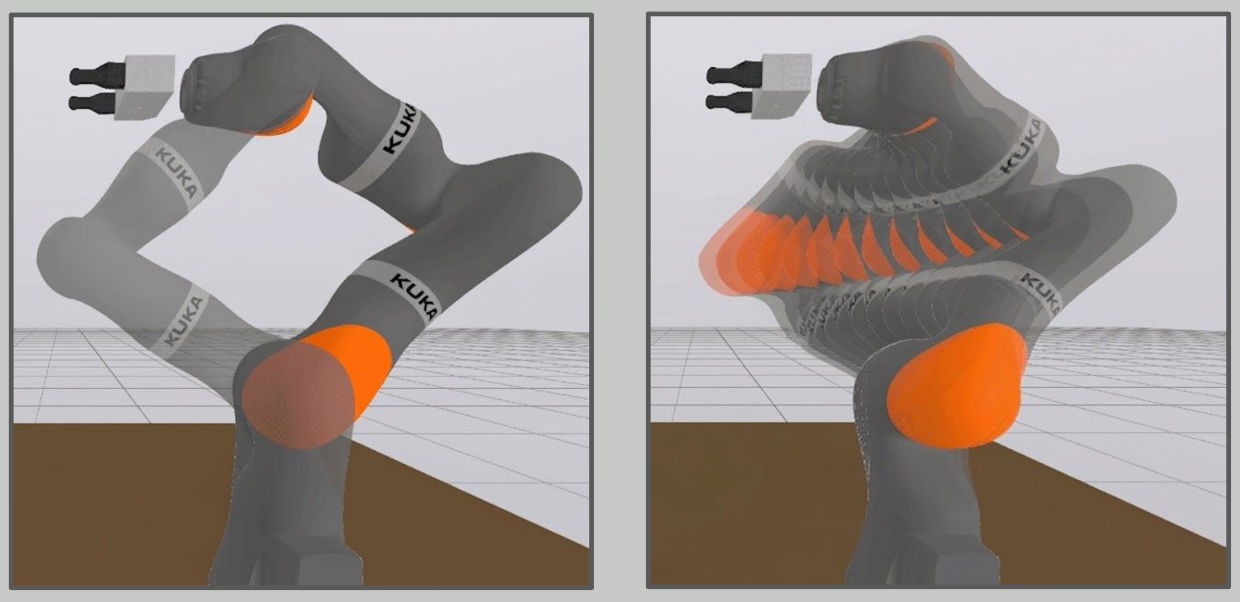
- *Joint angles are decision variables*
- *Constraints on end-effector (EE) pose*
- *Immense modeling flexibility*
- *No guarantees*

Analytic Inverse Kinematics

- *Symbolically-derived mapping from end-effector to joint angles*
- *Limited modeling flexibility*
- *Returns all solutions in O(1) time*

Self-Motion Parameters

- *There are many IK solutions for one EE pose*
- *"Self-motion parameters" disambiguate solutions*
- *Write* $\mathrm{IK} : \mathrm{SE}(3) \times \Psi \times \mathscr{K} \to \mathbb{R}^d$
- Ψ *continuous,* $\mathscr{K}$ *discrete (fix* $\kappa \in \mathscr{K}$ *in practice)*



Discrete (left) and continuous (right) self-motions.

Old Formulation

$$\begin{aligned} \min_q \quad & f(q) \\ \text{s.t.} \quad & {}^W X^G(q) = {}^W X^G_{\text{desired}} \\ & q_{\text{lb}} \le q \le q_{\text{ub}} \\ & g(q) \le 0 \\ & h(q) = 0 \end{aligned}$$

Change of variables $\xi : ({}^W X^G, \psi) \mapsto q$

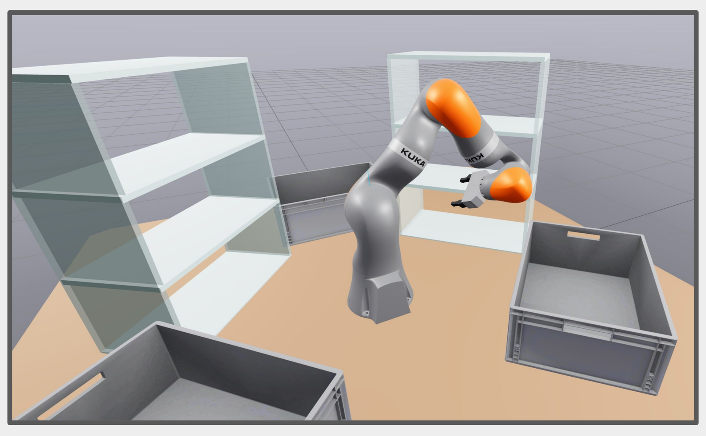
New Formulation

$$\begin{aligned} \min_{{}^W X^G, \psi} \quad & f(\xi({}^W X^G, \psi)) \\ \text{s.t.} \quad & \cancel{{}^W X^G = {}^W X^G_{\text{desired}}} \\ & q_{\text{lb}} \le \xi({}^W X^G, \psi) \le q_{\text{ub}} \\ & g(\xi({}^W X^G, \psi)) \le 0 \\ & h(\xi({}^W X^G, \psi)) = 0 \end{aligned}$$

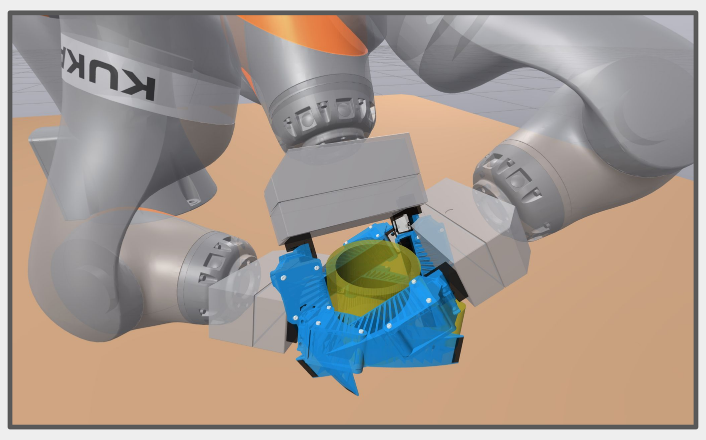
Reachability Constraints

- *Old style reachability:* $\mathrm{FK}(\mathrm{IK}(X, \psi)) = X$
  -> Active at all feasible points!
- *New style reachability:* $D_k(X) \le 0$
  -> Active only on boundary!
  -> Better for optimizers

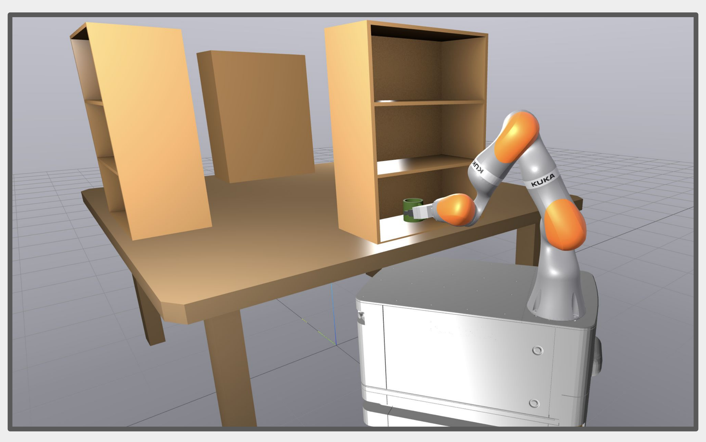
Experimental Setups



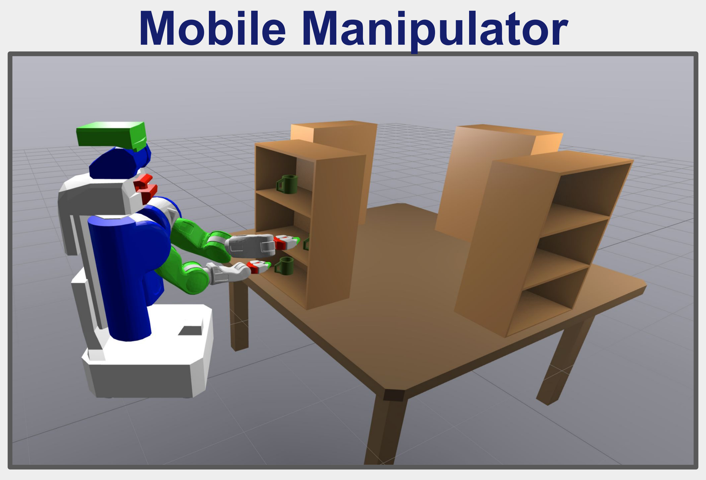
Arm on a Table



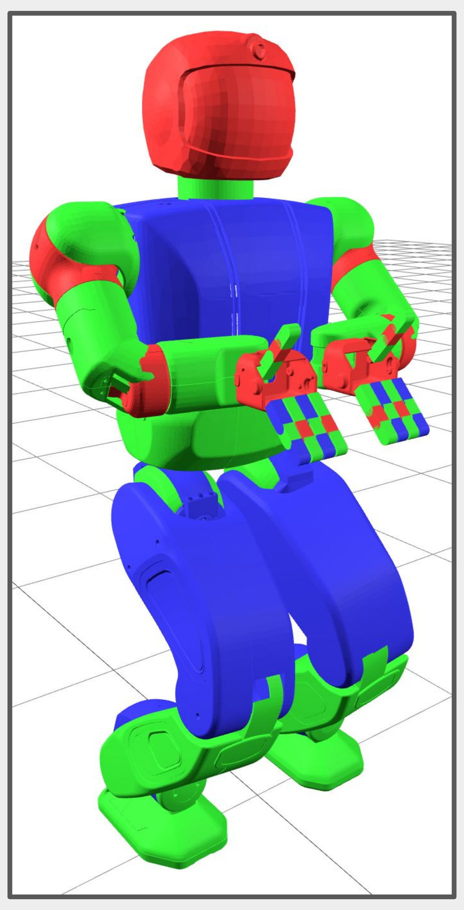
Grasp Selection



Mobile Manipulator



Bimanual Mobile Manipulator



Humanoid Stability

Numerical Results

Success Rate (Feasible Solution)

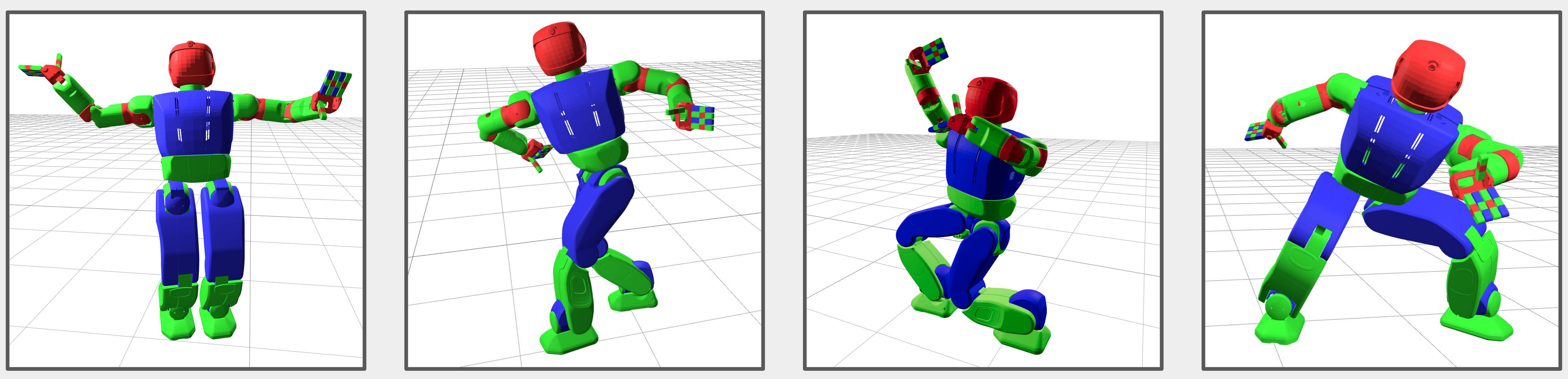
Experiment	d	d'	Interior Point (IPOPT)		Augmented Lagrangian (NLOPT)		Sequential Quadratic Programming (SNOPT)		Global-IK		Sampling
			New	Old	New	Old	New	Old	Fast	Precise	
Arm on a Table	7	1	**0.8951**	0.5392	**0.8733**	0.049	**0.9144**	0.463	0.88	0.97	1.00 ★
Grasp Selection	7	5	**0.9893** ★	0.9875	**0.47**	**0.47**	**0.7344**	0.6625	N/A	N/A	0.525
Mobile Manipulator	10	4	**0.7182**	0.5946	**0.405**	0.0074	**0.368**	0.2884	N/A	N/A	0.95 ★
Bimanual Mobile Manipulator	18	6	**0.7819** ★	0.5243	**0.68**	N/A	**0.3289**	0.2466	N/A	N/A	0.32
Humanoid Stability (Inequality)	31	19	**0.97** ★	0.70	**0.67**	N/A	N/A	N/A	N/A	N/A	N/A
Humanoid Stability (Equality)	31	19	**0.96** ★	**0.96** ★	**0.46**	N/A	N/A	N/A	N/A	N/A	N/A

Runtime (Seconds)

Experiment	Interior Point (IPOPT)		Augmented Lagrangian (NLOPT)		Sequential Quadratic Programming (SNOPT)		Global-IK		Sampling
	New	Old	New	Old	New	Old	Fast	Precise	
Arm on a Table	0.14 (0.11)	**0.07 (0.04)**	0.98 (0.84)	**0.24 (0.16)**	0.025 (0.025)	**0.02 (0.02)** ★	289	3362	0.101
Grasp Selection	0.45 (0.46)	**0.15 (0.15)** ★	2.68 (3.84)	**0.81 (1.13)**	1.07 (1.47)	**0.36 (0.26)**	N/A	N/A	1.69
Mobile Manipulator	0.68 (0.52)	**0.17 (0.12)** ★	1.37 (3.52)	**0.23 (0.32)**	0.90 (1.17)	**0.18 (0.17)**	N/A	N/A	0.60
Bimanual Mobile Manipulator	1.33 (0.64)	**0.40 (0.16)**	**6.01 (4.76)**	N/A	2.25 (2.30)	**0.18 (0.09)** ★	N/A	N/A	1.73
Humanoid Stability (Inequality)	**1.26 (1.12)** ★	3.45 (1.82)	**25.82 (17.82)**	N/A	N/A	N/A	N/A	N/A	N/A
Humanoid Stability (Equality)	**1.61 (1.31)** ★	1.65 (1.50)	**36.37 (24.00)**	N/A	N/A	N/A	N/A	N/A	N/A

Optimal Cost

Experiment	Interior Point (IPOPT)		Augmented Lagrangian (NLOPT)		Sequential Quadratic Programming (SNOPT)		Global-IK		Sampling
	New	Old	New	Old	New	Old	Fast	Precise	
Arm on a Table	**14.1**	14.6	14.3	**11.3**	**14.2**	15.0	5.5 ★	5.8	6.1
Grasp Selection	9.22	**5.83**	10.9	**3.6** ★	**10.9**	14.0	N/A	N/A	7.44
Mobile Manipulator	8.46	**7.28**	8.62	**3.44** ★	10.8	**6.81**	N/A	N/A	14.58
Bimanual Mobile Manipulator	35.2	**25.5** ★	**30.6**	N/A	37.7	**29.8**	N/A	N/A	43.8



Stable Postures for a Humanoid while Reaching a Given Hand Pose Target