Sampling-Based Motion Planning with Discrete Configuration-Space Symmetries

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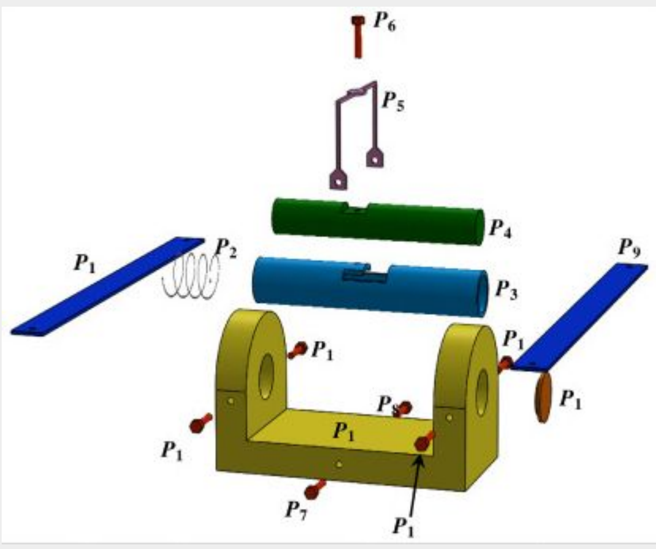
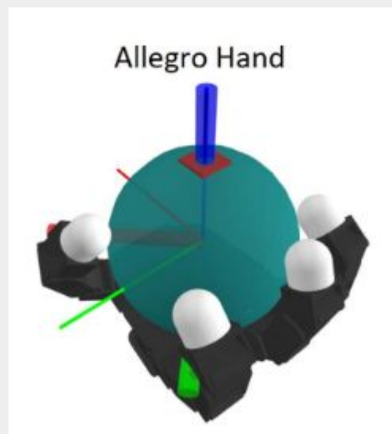
Configuration Spaces with Symmetries



Object Assembly Planning [1]



Contact Rich Manipulation [2]

Discrete vs Continuous Symmetries

- *Can drop dimensions for continuous symmetries*
  - *Sphere ->* $\mathbb{R}^3$
  - Circular Cone -> $\mathbb{R}^3 \times S^2$
- *Discrete symmetries maintain dimension*
- *Yields a **complicated configuration space topology***
- *Represent as Quotient Manifold Q/G*
  - *(For configuration space Q, symmetry group G)*

Primitives for Planning on Manifolds [3]

- ***Uniform Sample:** Uniformly sample Q, project*
- ***Local Sample:** Given* $[q] \in Q/G$*, uniformly sample from* $B_r(q)$ *and project, for any* $q \in [q]$.
- ***Distance Metric:** Given* $[q_1]$, $[q_2] \in Q/G$, $d([q_1],[q_2])$ *is the distance (in Q) between the closest representatives* $q_1 \in [q_1]$ *and* $q_2 \in [q_2]$.
- ***Local Planner:** Given* $[q_1],[q_2] \in Q/G$, follow the minimizing geodesic and check for collisions.

Geometry of *Q/G*

- $\mathrm{Vol}(Q/G) = \mathrm{Vol}(Q)/|G|$
- $r_{inj}(Q/G) \geq r_{inj}(Q)/|G|$
- *Definition: a path* $\gamma:[0,1]\to Q$ *is **δ-clear** if every point on* γ *is the center of a collision-free ball of radius* δ.
- If $\delta < r_{inj}(Q/G)$, then $\pi \circ \gamma:[0,1]\to Q/G$ is also **δ-clear**

Sample Complexity Improvements

- [4] *RRT algorithm (fewer iterations k)*

$$\mathbb{P}[\text{Not Reached in } k \text{ Iterations}] \leq \frac{1}{(m-1)!} k^m m e^{-|G|pk}$$

- [5] *PRM algorithm (fewer samples N)*

$$\mathbb{E}[N] \leq \frac{H\left(\frac{2\ell}{\delta}\right)\mathrm{Vol}(\mathcal{Q}_{\text{free}})}{|G|\,\mathrm{Vol}(B_{\delta/2}(\cdot))}$$

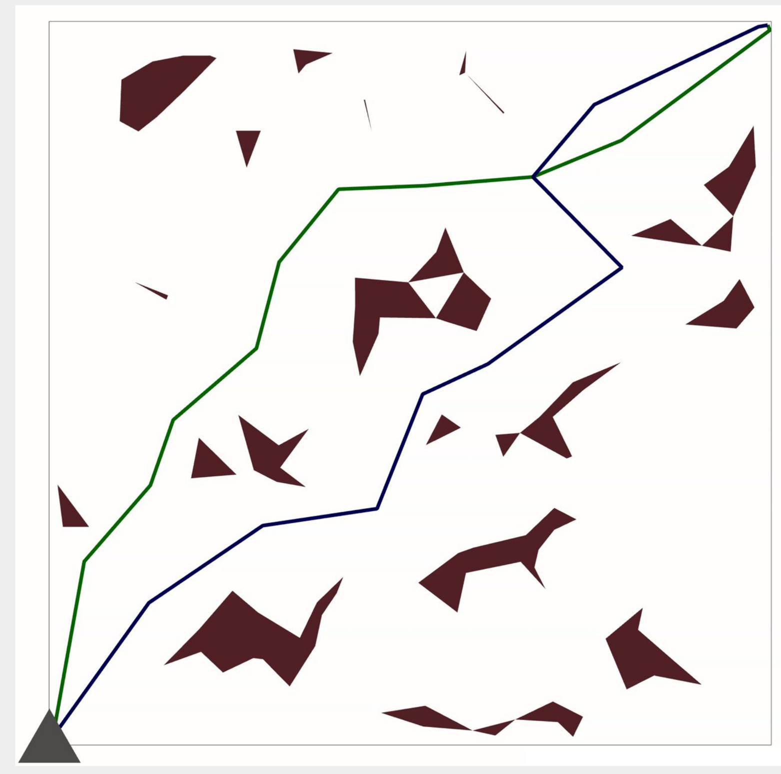
- [6] *PRM* algorithm (smaller connection radius ρ)*

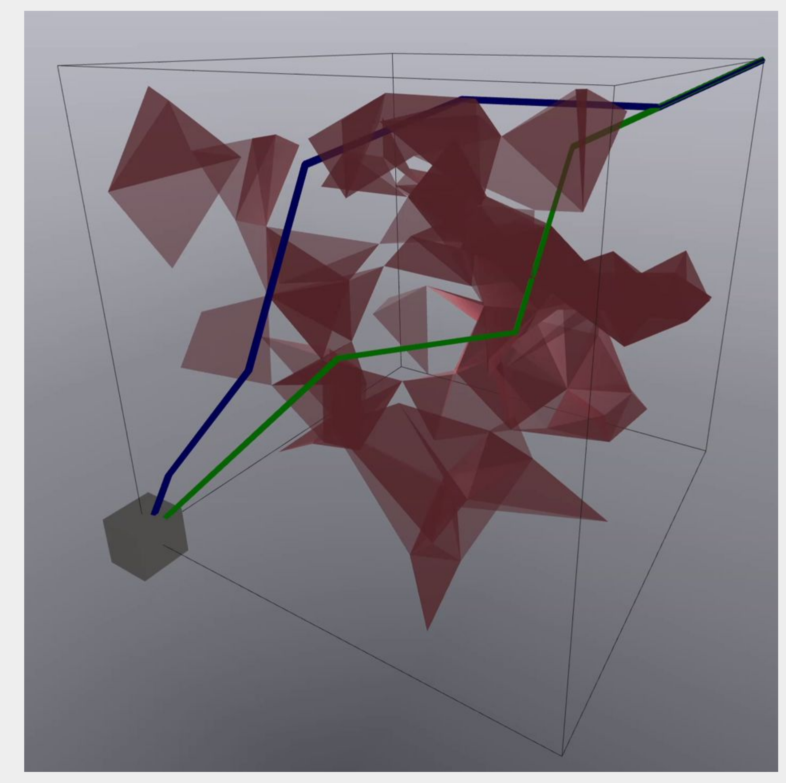
$$\rho_{\text{prm}} > 2(1+1/d)^{1/d}\left(\frac{\mathrm{Vol}(\mathcal{Q}_{\text{free}})}{\mathrm{Vol}(B_1(\cdot))}\right)^{1/d}|G|^{-1/d}$$

- [7] *RRT* algorithm (smaller connection radius ρ)*

$$\rho_{\text{rrt}} \geq (2+\theta)\left(\frac{(1+\epsilon/4)c^*\,\mathrm{Vol}(\mathcal{Q}_{\text{free}})}{(d+1)\theta(1-\mu)\,\mathrm{Vol}(B_1(\cdot))}\right)^{\frac{1}{d+1}}|G|^{\frac{-1}{d+1}}$$

Experimental Domains





2D path planning (left) and 3D path planning (right). Green path is symmetry-aware, blue path is symmetry-unaware.

Experimental Results

- *For each symmetric object, generate 10 random worlds, and 100 start/goal pairs per world -- **total of 24000 plans***
- *Compare to symmetry-unaware baselines*
- *Two sets of experiments*
  - *Equal resources*
  - *Reduced resources for symmetry-aware planners (informed by theoretical results)*
- *Summary of results (equal resources)*
  - ***RRT: 2-3x speedup, 1.1-1.4x shorter paths***
  - *RRT*: 1.1-1.6x slower, ~1.1x shorter paths*
  - *KNN-PRM*: 1.1-10x slower, ~1.1x shorter paths*
  - *|G| does not affect results*
- *Summary of results (unequal resources)*
  - *RRT*: 1.1-1.6x slower, ~1.1x shorter paths*
  - ***KNN-PRM*: 2-16x speedup** (grows with |G|), comparable paths*
  - ***RNN-PRM*: 2-32x speedup** (grows with |G|), comparable paths*
  - *Larger |G| leads to larger speedup for PRM methods*
- *Dimension scaling experiments: joint planning for multiple objects (up to 15 dimensions)*
  - Runtime improvement grows with dimension
  - Path lengths consistently better

Conclusions

- *Describe key facts of configuration space geometry*
- *Theory suggests better algorithmic performance*
- *Comprehensive experiments verify theory*

References

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[2] *Global planning for contact-rich manipulation via local smoothing of quasi-dynamic contact models*, Pang, Suh, Yang, and Tedrake 2023.
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[4] *Probabilistic completeness of RRT for geometric and kinodynamic planning with forward propagation*, Kleinbort, Solovey, Littlefield, Bekris, and Halperin 2018.
[5] *Measure theoretic analysis of probabilistic path planning*, Ladd and Kavraki 2004.
[6] *Sampling-based algorithms for optimal motion planning*, Karaman and Frazzoli 2011.
[7] *Revisiting the asymptotic optimality of RRT*, Solovey, Janson, Schmerling, Frazzoli, and Pavone 2020.