# Topologically-Informed Atlas Learning

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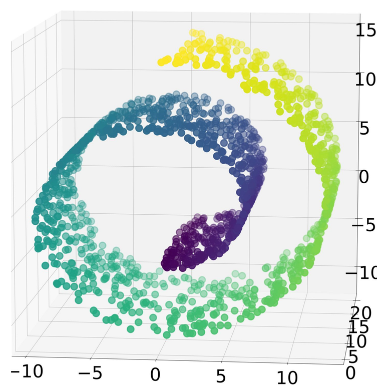
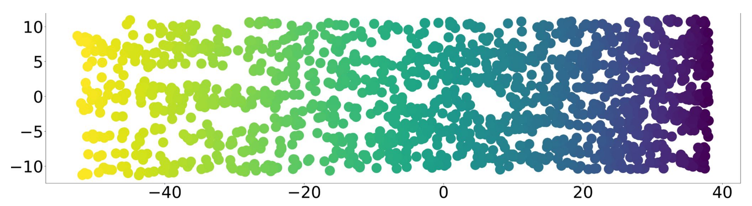
## Key idea: Estimating data topology improves manifold learning

## Introduction

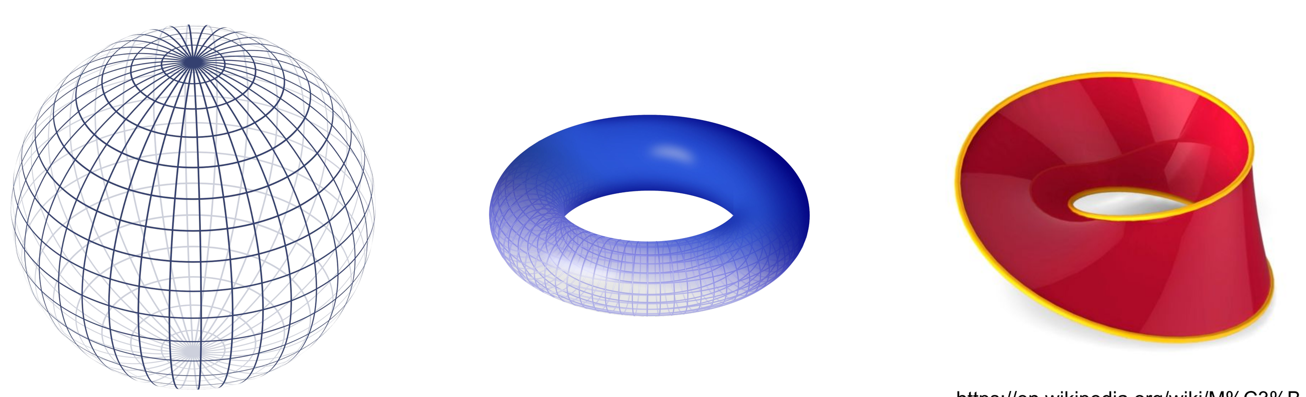
If high dimensional data lies along some smooth manifold, it can be "unwrapped" or "flattened" to achieve dimensionality reduction. This process is called manifold learning.





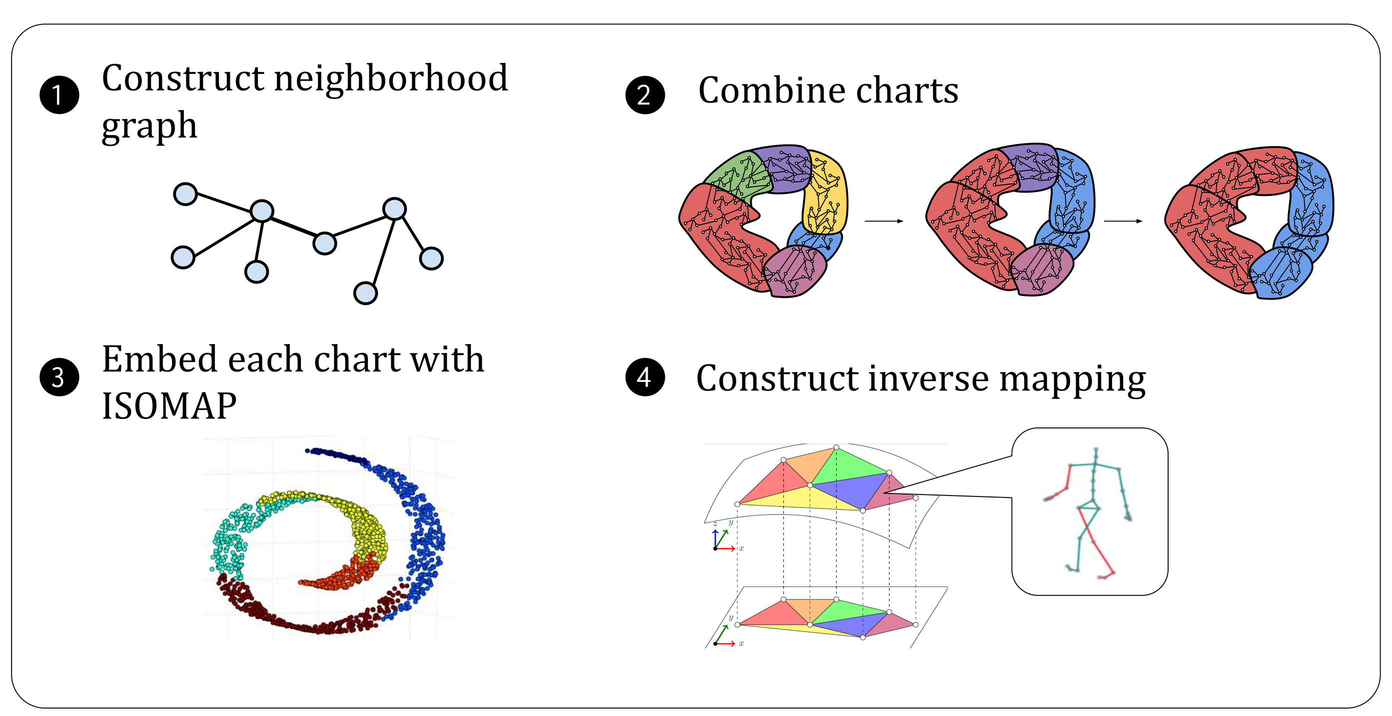
Embedding produced with ISOMAP [1]

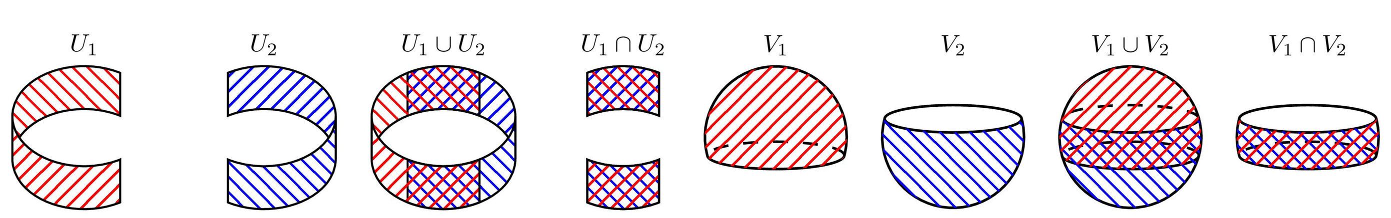
If the manifold is not topologically trivial, this approach may be impossible. For example, the sphere, torus, mobius band, SO(3), and SE(3) all cannot be flattened out. Instead, learn an atlas of coordinate charts [2][3]. We use an atlas to process human motion capture data and learn kinematic models for articulated objects [4].



https://en.wikipedia.org/wiki/N-sphere https://en.wikipedia.org/wiki/Torus https://en.wikipedia.org/wiki/M%C3%B6bius_strip

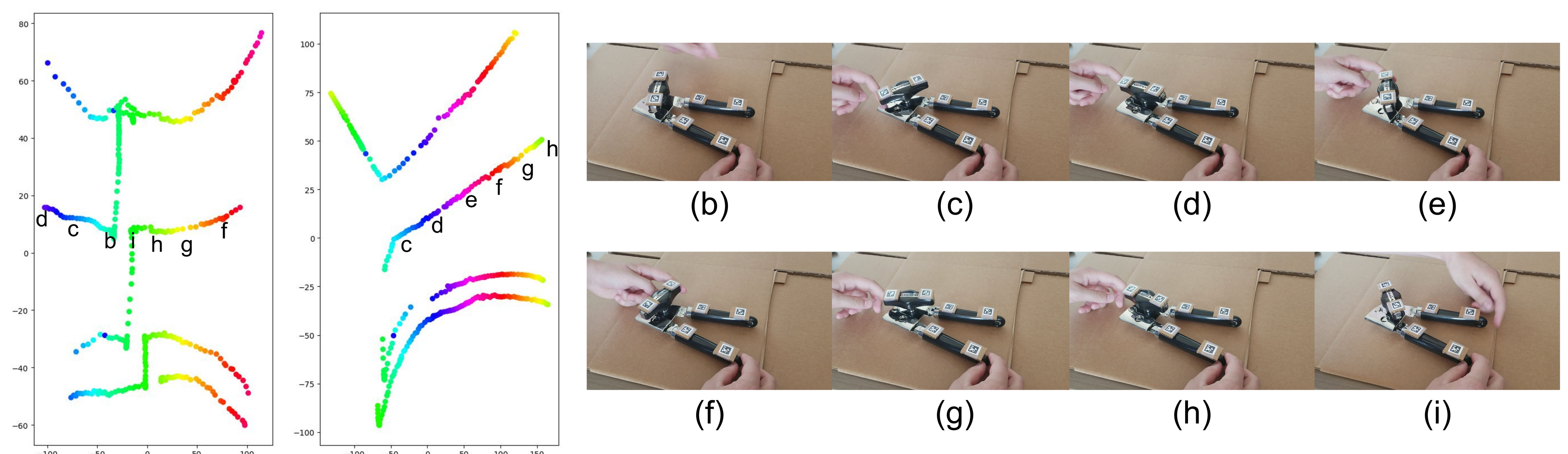
## Methodology



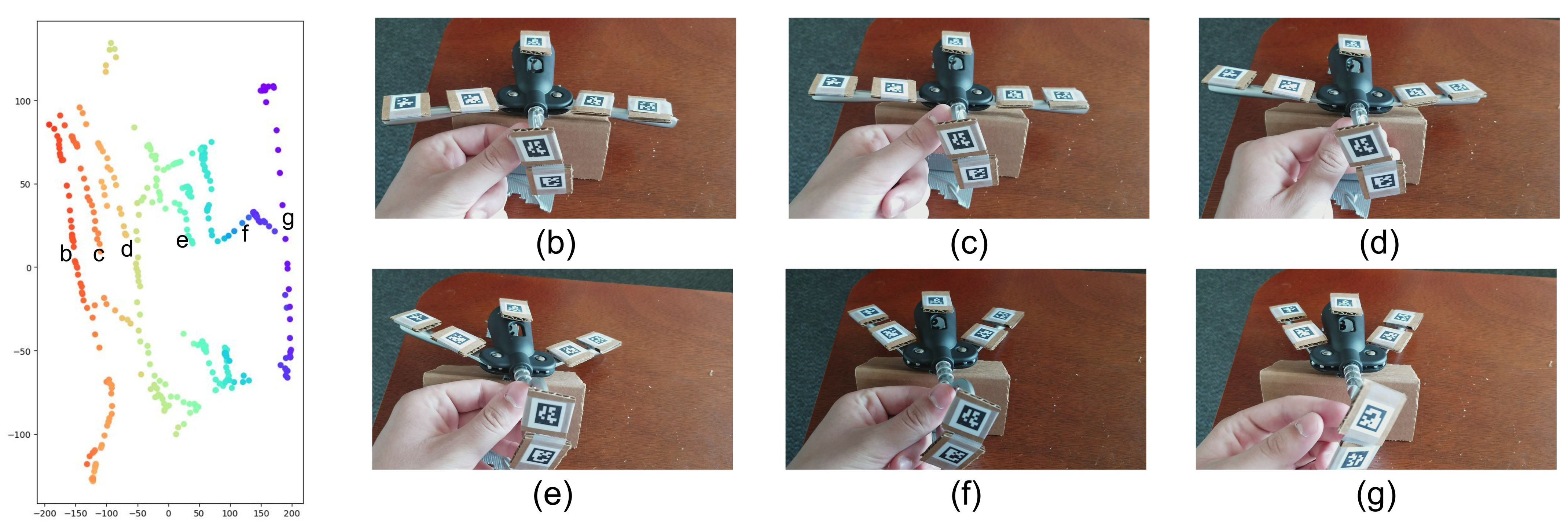




Exemplar cases where two charts cannot be combined without introducing a topological hole. $U_1 \cap U_2$ is disconnected, and $V_1 \cap V_2$ contains a hole. Holes are identified by searching for large atomic cycles [5].

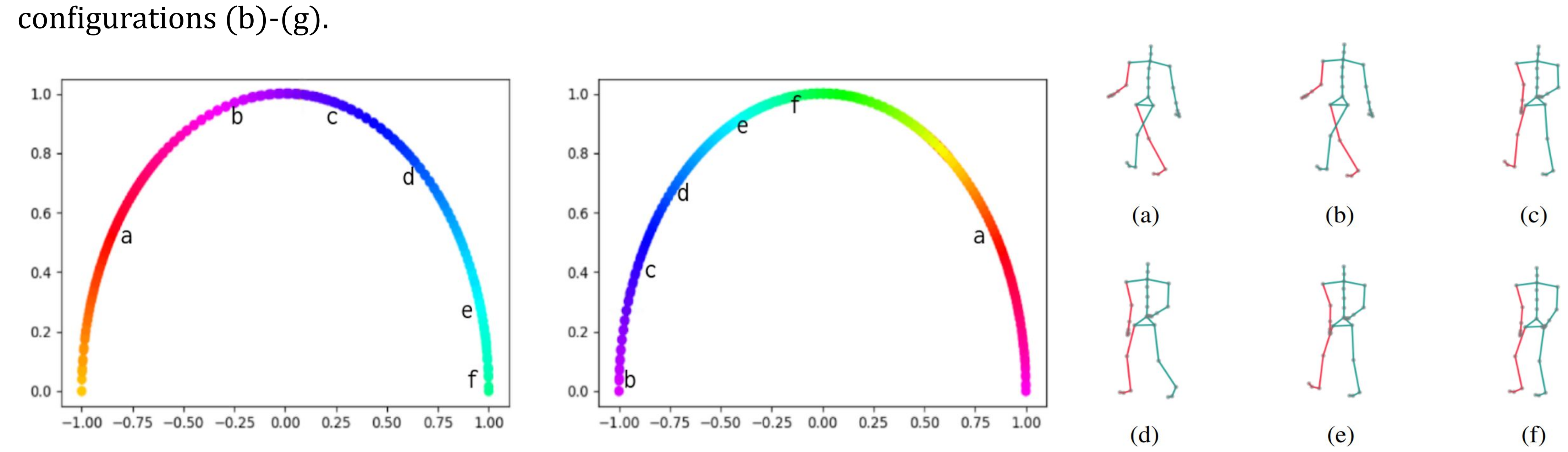
## Results




Learned kinematic model for a can opener (left), annotated with demonstrative object configurations (b)-(i).

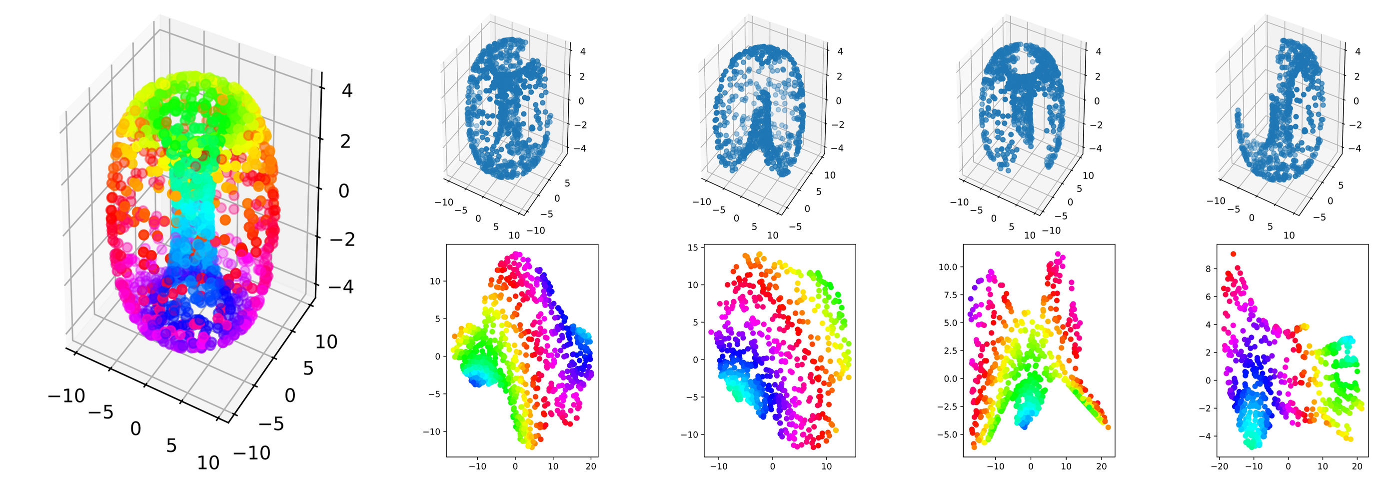



Learned kinematic model for a corkscrew bottle opener (left), annotated with demonstrative object configurations (b)-(g).




Learned embedding of motion capture data of a human walking, annotated with the embedding location of demonstrative poses.

## Results (Continued)



Atlas learning for data sampled from a torus (left). Our method constructs four charts, and we show each chart domain with its corresponding embedding.

Embedding Trustworthiness

| Experiment | ISOMAP | Autoencoder (4 Charts) Worst | Autoencoder (4 Charts) Mean | Autoencoder (15 Charts) Worst | Autoencoder (15 Charts) Mean | Our Atlas Learning Worst | Our Atlas Learning Mean |
|---|---|---|---|---|---|---|---|
| Sphere | 0.844 | **0.999** | **0.999** | **0.998** | **0.999** | **0.999** | **0.999** |
| Torus | 0.916 | 0.900 | 0.916 | 0.820 | 0.898 | **0.996** | **0.998** |
| Mobius Strip | 0.995 | 0.996 | 0.997 | 0.944 | 0.988 | **0.998** | **0.999** |
| Mocap | 0.991 | 0.782 | 0.858 | 0.829 | 0.914 | **0.996** | **0.996** |
| Can Opener Kinematic Model | 0.953 | 0.351 | 0.536 | 0.316 | 0.517 | **0.996** | **0.997** |
| Bottle Opener Kinematic Model | 0.985 | 0.46 | 0.686 | 0.243 | 0.491 | **0.993** | **0.993** |

Numerical accuracy (according to the manifold trustworthiness metric [6]) of the embeddings produced by ISOMAP [5], an atlas learning autoencoder [2], and our method. Best results for each experiment are bolded.

## Acknowledgements

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